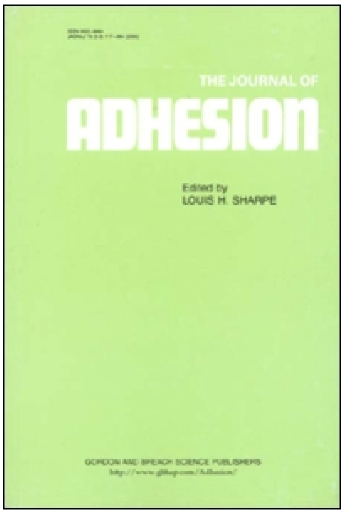

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The Stress-Strain Behaviour of Axially-Loaded Butt Joints

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The effects of the stress distribution in the adhesive layer of axially loaded butt joints have been examined theoretically and experimentally. It has been shown that the measured stress–strain behaviour of a butt joint is dependent on the triaxial stress state induced in the adhesive by the restraint of the adherends. This causes a butt joint to yield at a stress which is greater than the uniaxial yield stress of the adhesive. Conversely, the presence of stress concentrations can cause a butt joint to fail at a lower tensile stress than the failure stress of a bulk specimen tested in uniaxial tension. Therefore, the relationship between the strength of a butt joint and that of a bulk specimen of the same adhesive depends on the ductility of the adhesive. Furthermore, if the adhesive obeys a pressure-dependent yield criterion, the compressive yield stress of a butt joint can be much greater than the tensile yield stress of a similar joint and, under some circumstances, compressive yielding of a butt joint may be suppressed completely.

1. INTRODUCTION

Butt joints have been commonly used for measuring the tensile properties of adhesives, solders and brazes. They are often preferred to bulk specimens because the adhesive (or other bonding material) is tested in the thin film form as used in the actual joint. However, stress–strain data obtained from this type of specimen is often difficult to correlate with those obtained from tests on bulk specimens.

The apparent Young's modulus obtained for the adhesive in a butt joint is usually different from that of the bulk material. Volkersen[1] quoted experimental work by Müller[2] which showed that the Young's modulus of an adhesive apparently decreased as the thickness of the adhesive in the butt joint was increased. Similarly, Franzblau and Rutherford[3] obtained a higher value of Young's modulus for a thin film of adhesive than when it was tested in the bulk form. This apparent increase in Young's modulus is caused by the circumferential and radial stresses induced in a butt joint by the transverse restraint imposed on the adhesive by the much stiffer adherends. Adams and Coppendale[4] showed that, if this effect is taken into account, values of Young's modulus obtained from adhesive butt joints agreed well with the values obtained from bulk specimens.

It is also common for the tensile strength of an adhesive butt joint to be different from the uniaxial tensile strength of the adhesive measured in the bulk form. For example, Jennings[5] found that the strength of a particular epoxy resin in a butt joint was different from the tensile strength of bulk specimens of the same material measured by Ishai.[6] At a temperature of 0°C, the butt joints were slightly weaker than the bulk specimens, but at 60°C the butt joints were twice as strong as the bulk specimens. Similar results were obtained by Lewis and Ramsey[7] for another epoxy resin. The strength of butt joints also depends on the surface finish of the adherends. It is also known that brazed and soldered joints are often considerably stronger than the uniaxial tensile strength of the braze or solder from which they are made.[8]

The purpose of the work described here was to analyse the effects of the stress distributions in butt joints in an attempt to explain the apparent discrepancies between the strengths of butt joints and bulk specimens.

2. THEORY

Adams *et al.*[9] used the finite element method to analyse the stress distributions in adhesive butt joints loaded in tension, compression and torsion: a typical tensile (or compressive) stress distribution is shown in Figure 1. The bonded area comprises two different regions. In the central region, the direct stresses are uniform and the interfacial shear stress is zero. The radial and circumferential stresses were found to be essentially the same as those predicted by the analysis of Kuenzi and Stevens,[10] i.e.

$$\sigma_r = \sigma_\theta = \left[\nu_g - \frac{E_g \nu_a}{E_a}\right]\left[\frac{\sigma_z}{1-\nu_g}\right].$$

Around the periphery of the joint, there is a region in which the direct stresses are dependent on the radius and in which an interfacial shear stress is present. It is this shear stress which tends to restrain radial displacement of the adhesive and which induces the radial and circumferential stresses in the central region of the joint. In the peripheral region, there is also a variation of stress across the adhesive thickness. On the mid-plane of the adhesive, the direct stresses decrease to low values at the free surface (zero in the case of the radial stress) and the shear stress is always zero. On the adhesive–adherend interface, there is a stress concentration at the corner of the adherend. It seems quite probable that these stress concentrations could

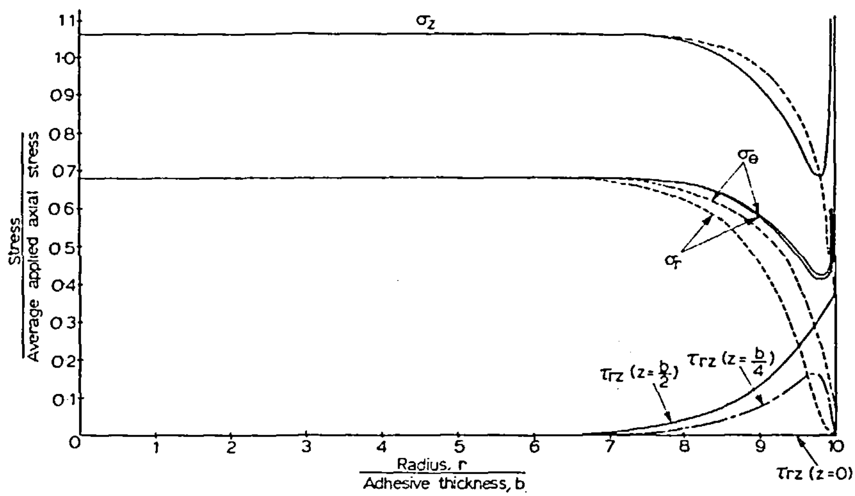

FIGURE 1 Stress distributions for a circular solid butt joint loaded in tension. —— $z = \pm b/2$ (interface); - - - $z = 0$ (mid-plane). Aspect ratio, i.e. ratio of bar diameter to adhesive layer thickness, is 20.

initiate failure in a brittle adhesive. As the highest stresses are at the interface, the surface preparation of the adherends is also likely to affect the strength of the joint if failure is initiated in this region.

For a particular value of adhesive Poisson's ratio, the radial width of the peripheral region (measured in terms of adhesive thicknesses) is independent of the diameter of the butt joint. For example, if the Poisson's ratio of the adhesive is 0.4, the peripheral region extends inwards approximately three adhesive thicknesses from the outside of the joint. Therefore, in a typical joint with a diameter approximately two orders of magnitude greater than the adhesive thickness, the stresses are uniform over a large proportion of the total bonded area.

Up to the failure load, the axial stress–strain behaviour of the butt joint will depend primarily on the response of the adhesive to the stress state in the central region of the joint, where the σ_r and σ_θ stresses are equal to each other but are less than the applied stress, σ_z. When a tensile load is applied to the joint, the stress state is equivalent to a uniaxial tensile stress, $\sigma_z - \sigma_r$, combined with a negative hydrostatic pressure, σ_r. Similarly, a compressive load applied to a butt joint induces a uniaxial compressive stress superimposed on a positive hydrostatic pressure.

Many metals obey a yield criterion which is independent of hydrostatic pressure (e.g. the Tresca or von Mises criteria). The Tresca criterion predicts that a material will yield if the resolved shear stress on any plane attains a critical value:

$$\tau_T = \tfrac{1}{2}(\sigma_{\max}-\sigma_{\min}) = \tfrac{1}{2}\sigma_T$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the maximum and minimum principal stresses, σ_T is the uniaxial yield stress and τ_T is the stress which would cause flow in pure shear. For most of the adhesive in a butt joint

$$\sigma_{\max} = \sigma_z$$

and

$$\sigma_{\min} = \left[\nu_g - \frac{E_g \nu_a}{E_a}\right]\left[\frac{\sigma_z}{1-\nu_g}\right] \simeq \left(\frac{\nu_g}{1-\nu_g}\right)\sigma_z$$

Thus, the Tresca criterion predicts that a butt joint should yield at a stress σ_y where

$$\sigma_y = \left(\frac{1-\nu_g}{1-2\nu_g}\right)\sigma_T$$

The von Mises criterion predicts that yield will occur if the elastic shear strain energy attains a critical value. In terms of the principal stresses, it can be expressed as

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 = 2\sigma_T^2$$

Using the same arguments as above, the von Mises criterion also predicts a butt joint to commence yielding at

$$\sigma_y = \left(\frac{1-\nu_g}{1-2\nu_g}\right)\sigma_T$$

The ratio σ_y/σ_T is plotted in Figure 2 for various values of Poisson's ratio, and is, of course, identical for both criteria.

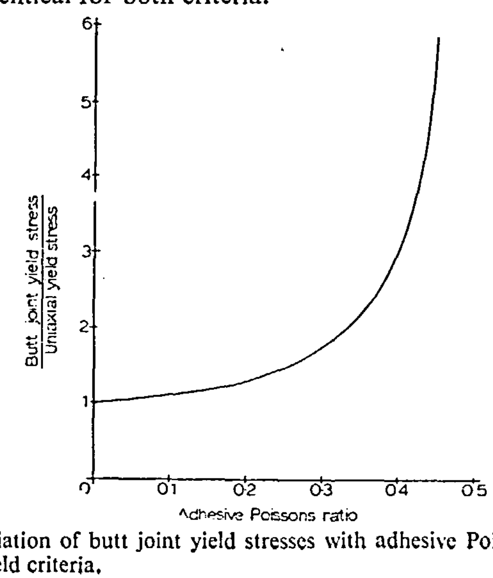

FIGURE 2 Variation of butt joint yield stresses with adhesive Poisson's ratio; Tresca and von Mises yield criteria.

The Tresca and von Mises yield criteria imply that the yield stress of a material is independent of the hydrostatic component of stress. Although this is true for many metals, the yield stress of many polymers has been found to depend on the hydrostatic component and, to describe their behaviour, various modifications to these criteria have been proposed.

Coulomb discovered that flow in certain granular materials occurred when the shear stress on any plane reached a critical value, τ_c, which varied linearly with the stress normal to that plane, σ_n. It is expressed as

$$\tau_c = \tau_c^0 - \mu_c \sigma_n$$

where τ_c° is the yield stress in shear when no normal stress is applied and μ_c is a constant for the material. It has since been applied to polymers by Whitney and Andrews[11] and by Bowden and Jukes,[12] among others. It can also be expressed as

$$\frac{\sigma_{\max}}{\sigma_T} - \frac{\sigma_{\min}}{|\sigma_c|} = 1$$

where σ_T and $|\sigma_c|$ are the absolute values of the yield stresses in uniaxial tension and compression respectively.

Bauwens[13] suggested a modification to the von Mises criterion to describe the behaviour of polyvinylchloride under a combination of shear and biaxial tension. It was applied to polymethylmethacrylate by Sternstein *et al.*[14] and by Bowden and Jukes.[15] In terms of the principal stresses it can be expressed as:

$$[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]^{\frac{1}{2}} = \frac{2\sqrt{2}|\sigma_c|\sigma_T}{|\sigma_c|+\sigma_T} - \frac{\sqrt{2}(|\sigma_c|-\sigma_T)(\sigma_1+\sigma_2+\sigma_3)}{|\sigma_c|+\sigma_T}$$

In three-dimensional principal stress space, the yield surface is a right circular cone with its axis coincident with the $\sigma_1 = \sigma_2 = \sigma_3$ line. In physical terms, the criterion proposes that the deviatoric shear stress causing yield in the material increases linearly with hydrostatic pressure. It should be noted that if $|\sigma_c| = \sigma_T$, the expression reduces to the more familiar von Mises criterion.

Another modification to the von Mises criterion was used by Raghava *et al.*[16] to describe the yield behaviour of polycarbonate and polyvinylchloride and was later applied to high density polyethylene by Raghava and Caddell.[17] In terms of the uniaxial tensile and compressive yield stresses, it can be written as:

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 = 2|\sigma_c|\sigma_T - 2(|\sigma_c|-\sigma_T)(\sigma_1+\sigma_2+\sigma_3)$$

It is similar to the previous criterion except that, instead of a cone, the yield surface is a three-dimensional parabola with its axis on the $\sigma_1 = \sigma_2 = \sigma_3$ line. It also reduces to the simple von Mises criterion if $|\sigma_c| = \sigma_T$.

Figure 3 shows sections through these yield surfaces in the $\sigma_2 \rightarrow \sigma_1$ vs. $\sqrt{2}\sigma_2$ ($= \sqrt{2}\sigma_3$) plane of principal stress space. It has already been shown that the σ_r and σ_θ stresses in most of the adhesive in an axially loaded butt joint are equal to each other and less than the applied stress, σ_z. Therefore, the stress state in an axially loaded butt joint can be represented as a point on a line in Figure 3 which passes through the origin. Increasing the axial stress results in the point moving from the origin along the line, the direction depending on whether the applied load is tensile or compressive. As the ratio of σ_r and σ_θ to σ_z increases with increasing Poisson's ratio, the angle between the line representing the butt joint stress state and the hydrostatic line ($\sigma_1 = \sigma_2 = \sigma_3$) will decrease. It will be noted in Figure 3 that, if a pressure dependent yield criterion is applicable, a substantial difference between butt joint tensile and

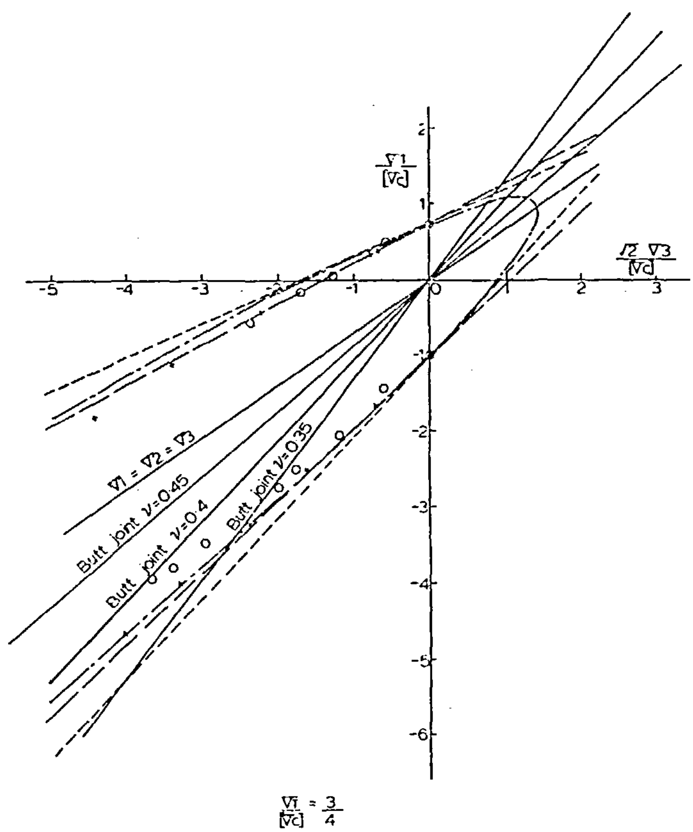

FIGURE 3 Pressure dependent yield criteria in principal stress space. ——— Mohr–Coulomb; - - - modified von Mises (conical), Refs. 13 and 14; —— – —— – modified von Mises (paraboloidal) Ref. 16. O, +, results from Ref. 18 for two epoxy resins.

compressive yield stresses can be expected for typical values of adhesive Poisson's ratio. This will be much greater than the difference between the uniaxial tensile and compressive yield stresses of the adhesive. Figure 4 shows the yield stresses of butt joints in tension and compression predicted by the three pressure dependent yield criteria plotted against adhesive Poisson's ratio. The ratio of the uniaxial compressive yield stress to the uniaxial tensile yield stress has been taken as 1.33 which is a typical value for polymers, including epoxy resins.[18] It should be noted that the yield stress of butt joints in compression is very sensitive to small changes in Poisson's ratio in the range 0.35 to 0.4 which are typical values for structural adhesives. The Poisson's ratio of many materials, including polymers,[19] increases as the material begins to yield. In the case of a butt joint, this will decrease the tendency for further yield to occur (see Figure 3). If the Poisson's ratio exceeds a certain value, the line representing the butt joint stress state in Figure 3 will never intercept the Mohr–Coulomb or the conical von Mises yield surfaces on the compressive side of the origin, and the butt joint will never even begin to yield under the influence of a compressive load.

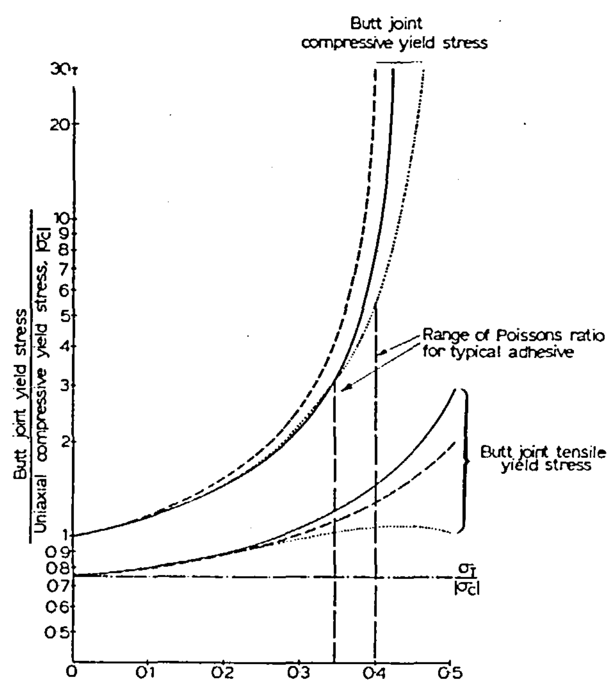

FIGURE 4 Variation with adhesive Poisson's ratio of butt joint yield stresses predicted by various yield criteria for $|\sigma_c|/\sigma_T = 1.33$. —— Mohr–Coulomb; - - - conical von Mises; . . . paraboloidal von Mises.

3. EXPERIMENTAL METHOD

3.1 Butt joints

Some preliminary tests were performed on butt joints bonded with BSL 308A film adhesive† to compare their stress–strain behaviour in tension and compression. Two specimens were tested, each consisting of two EN25 steel end-pieces, between which were bonded two rings of the same material, as shown in Figure 5. The adhesive film was cut to size and placed between the surfaces

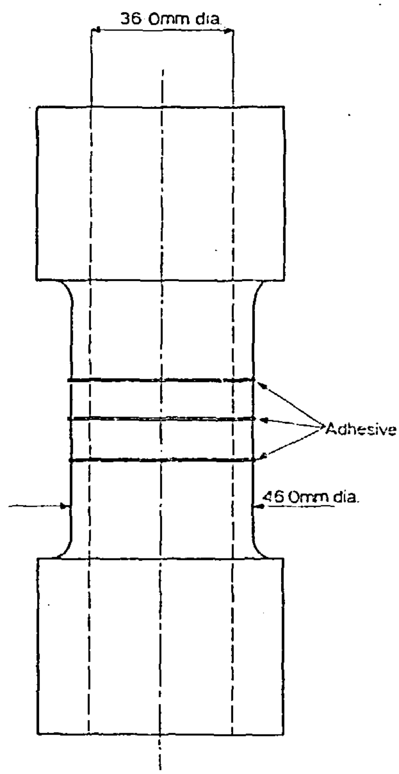

FIGURE 5 Butt joint specimen used in tension/compression tests with BSL308A adhesive.

to be bonded which had previously been grit-blasted and degreased in trichlorethylene vapour. The specimens were cured in a spring-loaded jig which was designed to maintain the alignment of the specimens and to apply an axial load until the adhesive layers had contracted to a thickness of 0.13 mm. The adhesive was cured for a total of 3 hours, which included 2 hours to

† All the adhesives used in the tests described here were manufactured by CIBA-GEIGY (UK) Ltd.

attain the recommended curing temperature of 150°C. Both specimens were first tested in compression using a pair of extensometers to measure the change in length of a 25.4 mm gauge length which included the three layers of adhesive. As the adhesive modulus was much less than that of the adherends, the combined deflection of the three layers of adhesive was of the same order of magnitude as the total deflection of the adherends between the extensometer knife edges. The adhesive strain was calculated by subtracting the extension of the adherends from the total recorded deflection. The tests were then repeated in tension. These tests were conducted quasi-statically, i.e. the extensometer reading was recorded between each increment of applied load.

A second series of butt joints was tested at constant strain rates on a servo-controlled testing machine. Two liquid epoxy resins (MY750 and AY103) were used which were also suitable for casting into bulk specimens. The MY750 was mixed with 85 parts per hundred by weight of hardener HY906 and 2 parts per hundred by weight of accelerator DY062. It was cured at 100°C for 3 hours which produced a rather brittle material. In contrast, AY103 is a plasticised epoxy which was mixed with 17 parts per hundred by weight of hardener HY956 and cured at 100°C for 3 hours. The 25.4 mm diameter HE30WP aluminium alloy adherends were grit-blasted, degreased in trichlorethylene vapour and etched in a mixture of chromic and sulphuric acids prior to bonding. Fifteen specimens were tested, seven containing MY750 and eight containing AY103. The strain across the 0.5 mm thick single layer of adhesive was continuously monitored by extensometers which incorporated linear variable differential transformers.

3.2 Bulk tensile specimens

Bulk specimens were machined from blocks of MY750 and AY103 which had been subjected to the same curing schedules as the butt joint specimens. The surfaces of the gauge lengths were polished with 1000 grade wet and dry emery paper using a motion parallel to the axis of the specimen. The fine scratches produced by the emery paper were then removed by a proprietary metal polish. The specimens were tested in tension at constant strain rates in the range 10^{-3}/min to 10^{-1}/min.

4. RESULTS

4.1 Butt joints

The BSL308A butt joint specimens were both loaded in compression to a stress of 340 MPa and, up to this stress, the load-deflection curves were linear. As no apparent damage had been caused by the compression tests, both specimens were subsequently loaded to failure in tension. Non-linearity was detected at a stress of approximately 20 MPa and the specimens failed at stresses of 78.8 MPa and 80.2 MPa respectively. The stress–strain curves are shown in Figure 6. The failure surfaces were predominantly cohesive (i.e. failure occurred within the adhesive layer rather than at either of the adhesive–adherend interfaces), although around the inner and outer edges of the annulus failure had occurred at the adhesive–adherend interface.

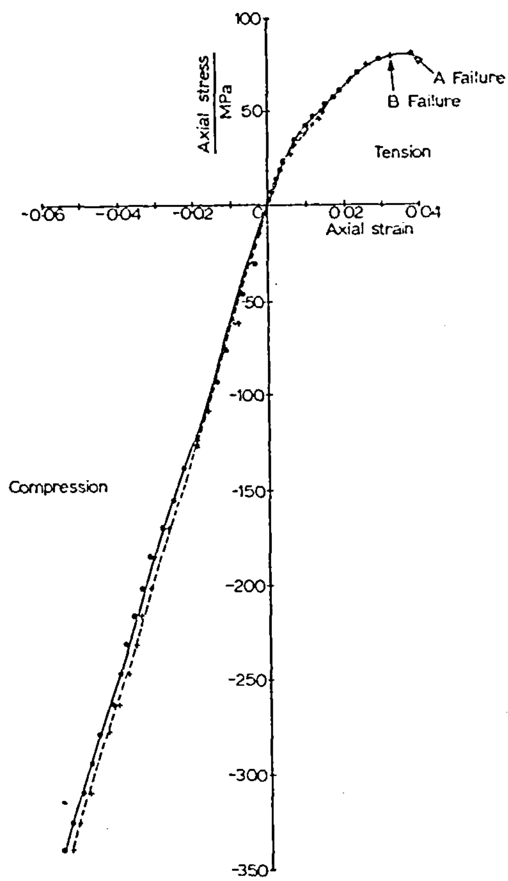

FIGURE 6 Adhesive stress–strain curves from BSL308A butt joint tests. —●— Specimen A; – – + – – Specimen B.

Two of the second series of butt joints (one MY750 specimen and one AY103 specimen) were loaded to a stress of 150 MPa in compression and, as with the BSL308A specimens, the stress–strain curves were linear. All the specimens were then tested in tension at adhesive strain rates of either

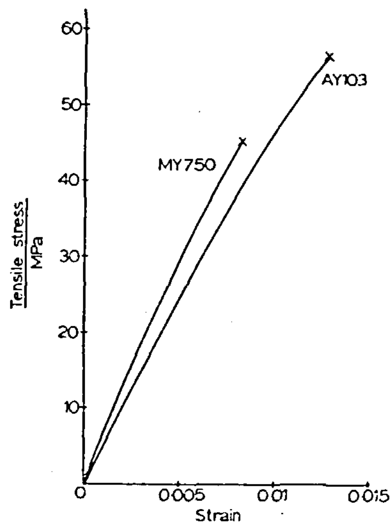

FIGURE 7 Typical adhesive stress–strain curves from AY103 and MY750 butt joint specimens.

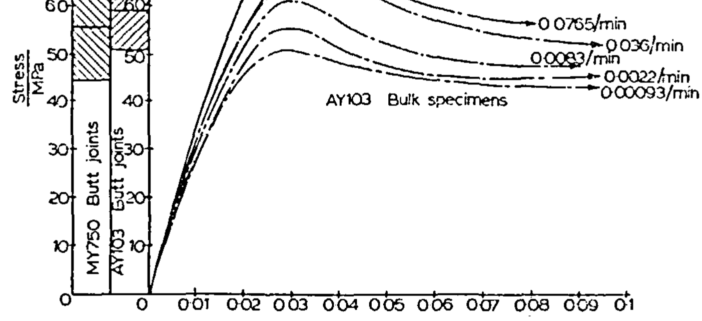

FIGURE 8 Adhesive tensile stress–strain curves from AY103 and MY750 bulk specimens. × specimen failed, →test interrupted. Also shown is the mean and range of the butt joint strengths for AY103 and MY750.

0.01/min or 0.001/min. The difference in strain rate did not have any significant effect on the strength of the specimens or on their stress–strain curves. Typical tensile stress–strain curves for the two adhesives are shown in Figure 7. Very little deviation from linear elastic behaviour was observed in any of the specimens. All the failure surfaces revealed a small area (typically 0.5 mm across) where failure had initiated at, or very close to, one of the interfaces. Over the remainder of the bonded area, cohesive failure had left adhesive attached to both adherends as before. The failure initiation sites in the AY103 specimens were distributed randomly over the bonded area, but, in all the MY750 specimens, failure had initiated within 0.5 mm of the circumference. The ranges and mean values of the failure loads for the MY750 and AY103 specimens are shown in Figure 8.

4.2 Bulk tensile specimens

The tensile stress–strain curves obtained from the MY750 and AY103 bulk specimens are also shown in Figure 8. The AY103 stress–strain curves all showed the same characteristics: following an initial linear region, the material gradually yielded until a maximum stress was reached. This was followed by a period of strain-softening until the material flowed at a lower level of stress. The tests were interrupted between strains of 0.08 and 0.1 as the extensometry was not capable of measuring beyond this. None of the AY103 specimens failed during these tests.

The behaviour of the unplasticised MY750 was different from the behaviour of the AY103 in several respects. All the MY750 specimens failed in a brittle manner at strains between 0.028 and 0.043 and the stress–strain curves did not pass through a maximum. The behaviour of the MY750 was less sensitive to strain rate than was the AY103: there was no correlation between the stress (or strain) at failure and strain rate.

5. DISCUSSION OF RESULTS AND CONCLUSIONS

The considerable difference between the tensile and compressive behaviour of the butt joints is in agreement with the predictions of Section 2 and suggests that the adhesive is obeying a pressure-dependent yield criterion. In compression, yielding was apparently suppressed, at least up to the comparatively high stress of 340 MPa in the case of the BSL308A and at least up to 150 MPa in the case of the other two adhesives. Some non-linear behaviour occurred in tension but, for two of the adhesives, failure appeared to initiate in the regions where the high tensile stress concentrations were predicted at the adherend corners.

The relation between the tensile strength of butt joints and the tensile strength of bulk specimens depends on several factors. Bulk specimens are unable to support a greater stress than the uniaxial yield stress of the material (as was the case with the AY103 bulk specimens). A brittle material may fail at a lower stress than the uniaxial yield stress (as was the case with the MY750 bulk specimens). However, in a butt joint, gross yielding is suppressed by the predominantly triaxial stress state in the adhesive, although some local yielding may occur at the stress concentrations around the edge of the joint. If the bulk specimens of a particular adhesive fail in a brittle manner, then the butt joints of the same material are likely to fail at an even lower stress because of the stress concentrations. This was observed with the MY750 butt joints. However, if the adhesive yields in a ductile manner in uniaxial tension, the butt joints may be stronger than the bulk specimens. This was observed by Jennings[5] for an epoxy resin at temperatures in excess of about 35°C: Lewis and Ramsey reported similar results.[7] The AY103 adhesive tested here happened to have a butt joint strength similar to its uniaxial tensile yield stress at the testing temperature (20°C). A more ductile adhesive could have a butt joint strength considerably higher than its uniaxial yield stress. This explains why brazed joints are often stronger than the bulk strength of the ductile braze material.[8]

It has been shown that the complex stress distributions in axially loaded butt joints make it difficult quantitatively to predict their stress–strain behaviour and ultimate tensile strength from the bulk properties of the adhesive without the use of a non-linear stress analysis and a detailed understanding of the response of the adhesive to the local stress concentrations around the perimeter of the joint. Conversely, it would be very difficult to use the stress–strain data obtained from axially loaded butt joints to predict the behaviour of an adhesive in a different type of joint (e.g. a lap joint) in which the stress distributions are likely to be completely different. Bulk specimens are not susceptible to the problems inherent in butt joint tests and are more suitable for providing reliable data on the response of adhesives to various known states of stress although care should be exercised in controlling the curing schedule. If a particular adhesive is not suitable for making into large bulk specimens, it should be possible to obtain uniaxial tensile stress–strain data by testing an unsupported thin film of the adhesive.

Despite the difficulty in making quantitative comparisons between the behaviour of an adhesive in a butt joint and its behaviour in the bulk form, this study has highlighted the causes of the discrepancies and has explained why butt joints can be either weaker or stronger than bulk specimens depending on their ductility, even if the adhesive in both types of specimen has been subjected to the same curing schedule.

NOTATION

E	Young's modulus
σ	direct stress
τ	shear stress
ν	Poisson's ratio

Suffixes

a	refers to adherend
g	refers to adhesive
r,θ,z	polar co-ordinates
1,2,3	principal stress directions
y	refers to yield

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